Broadband Signal Enhancement of Seismic Array Data: Application to Long-period Surface Waves and High-frequency Wavefields

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Abstract

We are developing a radically new approach to the problem of low signal-to-noise long-period surface waves using a newly developed approach that Thomson (1990) calls quadratic inverse theory. Analysis of surface wave dispersion requires precise measurements of relative phase. The double coherence estimate provides the the coherence and phase between different frequencies in the same signal. We tested the multitaper double-coherence technique with simple signals. The signals were designed to mimic the characteristics of idealized seismic data (both surface and body waves) but with defined frequency and amplitude. Initially, pure signals were processed and plotted. The signals were then degraded by adding random noise and plotted. We compared the results from the pure signals and the noisy signals to identify robust features in the coherence plots. Tests with real seismic data yield patterns similar to those seen in synthetic tests. We plan to conduct further tests with seismic data and investigate methods to further improve signal resolution.

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ENHANCEMENT OF SURFACE WAVES WITH A BROADBAND ARRAY

The most successful technique yet devised for the discrimination of nuclear explosions and earthquakes is the now standard M_S versus m_b technique. Unfortunately, this technique fails for low-yield nuclear tests when the signal-to-noise of the long-period surface wave train drops to a low enough level to make measurement of M_S impossible. We are developing a radically new approach to this problem using a newly developed approach that Thomson (1990) calls quadratic inverse theory. Analysis of surface wave dispersion requires precise measurements of relative phase. However, a classical problem in time series analysis is that the standard methods of estimating spectra and autocorrelations loose phase information. Phase measurements using higher order statistics such as the bispectrum assume stationarity and typically have higher variances than second order methods. They are also much less robust than standard estimates and are more susceptable to noise.

The fundamental difference between stationary and non-stationary signals is that in a non-stationary signal energy at different frequencies are correlated. We are attempting to find the phase using some properties of non-stationary signals combined with some new estimation methods. The double coherence estimate provides the the coherence and phase between different frequencies in the same signal. We are testing this technique as a additional tool to refine dispersion curves for smaller events.

Multitaper double coherence

The double coherence technique uses spectral estimates derived from the multitaper method (Thomson, 1982; Park *et al*, 1987) to determine cross spectral coherences of two signals. Double coherence differs from standard estimates of coherence in that coherence is calculated between all the frequencies in the two signals rather than at one frequency.

The multitaper spectral estimates combines weighted spectral estimates from a number of differently tapered versions of a given signal to yield a high-resolution spectral estimate. The tapers are mutually orthogonal and are based on the prolate spheroidal tapers developed by Slepian (1983). Multitaper estimates have the advantage that they are optimized to minimize leakage from outside the desired band and provide statistically independent estimates of the spectra. Each spectral estimates (S_{ij}) is determined using:

$$S_{ij}(f) = \frac{A}{K} \frac{\sum_{k=0}^{K-1} \lambda_k d_k^2(f) y_{ik}(f) y_{jk}(f)^*}{\sum_{k=0}^{K-1} d_k^2(f)}$$

where d_k are the weights of each spectral estimate, λ_k are the eigenvalues associated with each taper, and y_k are the discrete Fourier transforms of the tapered data. A is determined using $A = \sum_{k=0}^{K-1} \lambda_k^{-1}$ and K is the number of tapers.

The multitaper double coherence $\gamma_{ij}^2(f)$ is calculated using

$$\gamma_{ij}^2(f_{1,2}) = \frac{|S_{ij}(f_1)S_{ij}(f_2)^*|}{S_{ii}(f_1)S_{jj}(f_2)}$$

This corresponds to standard coherence in the case where $f_1 = f_2$ which is the diagonal of the double coherence estimate. Since the numerator is in effect multiplying a signal by a time-delayed version of the frequency components of the other signal, this estimator is sensitive to dispersive signals.

Tests with synthetic data

We tested the multitaper double-coherence technique with simple signals. The signals were designed to mimic the characteristics of idealized seismic data (both surface and body waves) but with defined frequency and amplitude. Initially, pure signals were processed and plotted. The signals were then degraded by adding random noise and plotted. We compared the results from the pure signals and the noisy signals to identify robust features in the coherence plots.

The amplitude and phase coherence of these signals is plotted as two square matrices, with the frequencies (normalized to the Nyquist frequency) of one signal on the x axis and the frequencies of the other signal on the y axis. The intersection of each frequency shows the cross-spectral coherence of the two signals. The auto-coherence of white noise (generated by an random number generator), for example, shows a diagonal line on both the amplitude and phase plots since the coherence of a signal with itself at a given frequency is unity and zero-phase. We also plotted the cross-diagonal of the phase coherence plot to show the phase relationship between different frequencies.

Varying-frequency signals. A sine wave that varies in frequency, typical of dispersed surface waves, shows a clear and characteristic pattern (Figure 1). The pattern is dominated by a zone of high coherence very near the diagonal and a larger patch of high coherence located above the intersection of the two frequency ranges. A broad zone of low coherence, less visible on Figure 1, is present at the intersection of the two frequency ranges. This patch of low coherence is present where the energy in the frequency band is high but the sum of the signal multiplied with its conjugate is low, producing a low overall coherence amplitude. This zone of low coherence is

most resistant to the addition of noise, since noise reduces coherence but doesn't greatly change the energy level in that frequency band.

The plots of phase coherence were seriously degraded by the addition of noise. We also tested modulating the sweep signal to produce 'packets' of varying waves similar to the effects of multi-pathed surface waves. This had little effect on the results. Tests of sweep signals with varying ranges showed that the coherence decreased as the range of frequencies increased, as expected. The zone of high coherence was reduced was the addition of noise but a high amplitude zone remained. Coherence was also decreased by increasing the range of frequencies, as expected, since two closely matched frequencies are more coherent that two separate ones.

Impulsive signals. Several time-limited signals, intended to represent body waves, were tested. Figure 2 shows the results for a sinc-like function. The pure signal had a very high amplitude coherence across most frequencies except for a band near the center frequency of the sine function. Since the amplitudes of the off-center frequencies were very low, the addition of noise reduced the coherence. The addition of noise actually improved the contrast in coherency level and a diffuse patch of higher coherency is observed centered around the frequency of the sin function. A Ricker wavelet showed similar results. This signal produced an especially clear pattern in the phase spectrum, as the components of an impulse signal are closely related in phase. This pattern is obvious in the cross-diagonal plot. Noise decreased the phase coherence but the basic trend is still evident.

Tests with real data. We tested the algorithm with seismic data recorded by the Kyrgyzstan broadband seismic network. Figure 3 shows some of the preliminary results. The upper two plots show coherence and phase for a teleseismic P arrival (and following phases) from an event 28° away. Although the time domain appearance differs greatly, the signal yields a pronounced phase pattern similar to the impulsive sinc-like function in Figure 2, probably due to the time-limited nature of the signal. Coherence of surface waves from a small regional (250 km) event in southern Kyrgyzstan is shown in the lower part of Figure 3. The mixed frequency content of the signal creates a complicated pattern of coherence at higher frequencies and a band of low coherence at very low frequencies.

Conclusions. We demonstrate the use of double coherence algorithm to resolve synthetic representations of seismic signals in the presence of noise. Tests with real seismic data yield patterns similar to those seen in synthetic tests. We plan to conduct further tests with seismic data and investigate methods to further improve signal resolution.

References.

Park, J. J., F. L. Vernon III, and C. R. Lindberg, Multitaper spectral analysis of high-frequency seismograms, *J. Geophys. Res.*, **92**, 12,675-12,684, 1987.

Slepian, D., Some comments of Fourier analysis, uncertainty, and modeling, SIAM Rev., **25**, 379-393, 1983.

Thomson, D. J., Spectrum estimation and harmonic analysis, IEEE Proc.,**70**, 1055-1096, 1982.

Thomson, D. J., Quadratic-inverse spectrum estimates: application to palaeoclimatology, *Phil. Trans. R. Soc. Lond. A.,* **332**, 539-597, 1990.
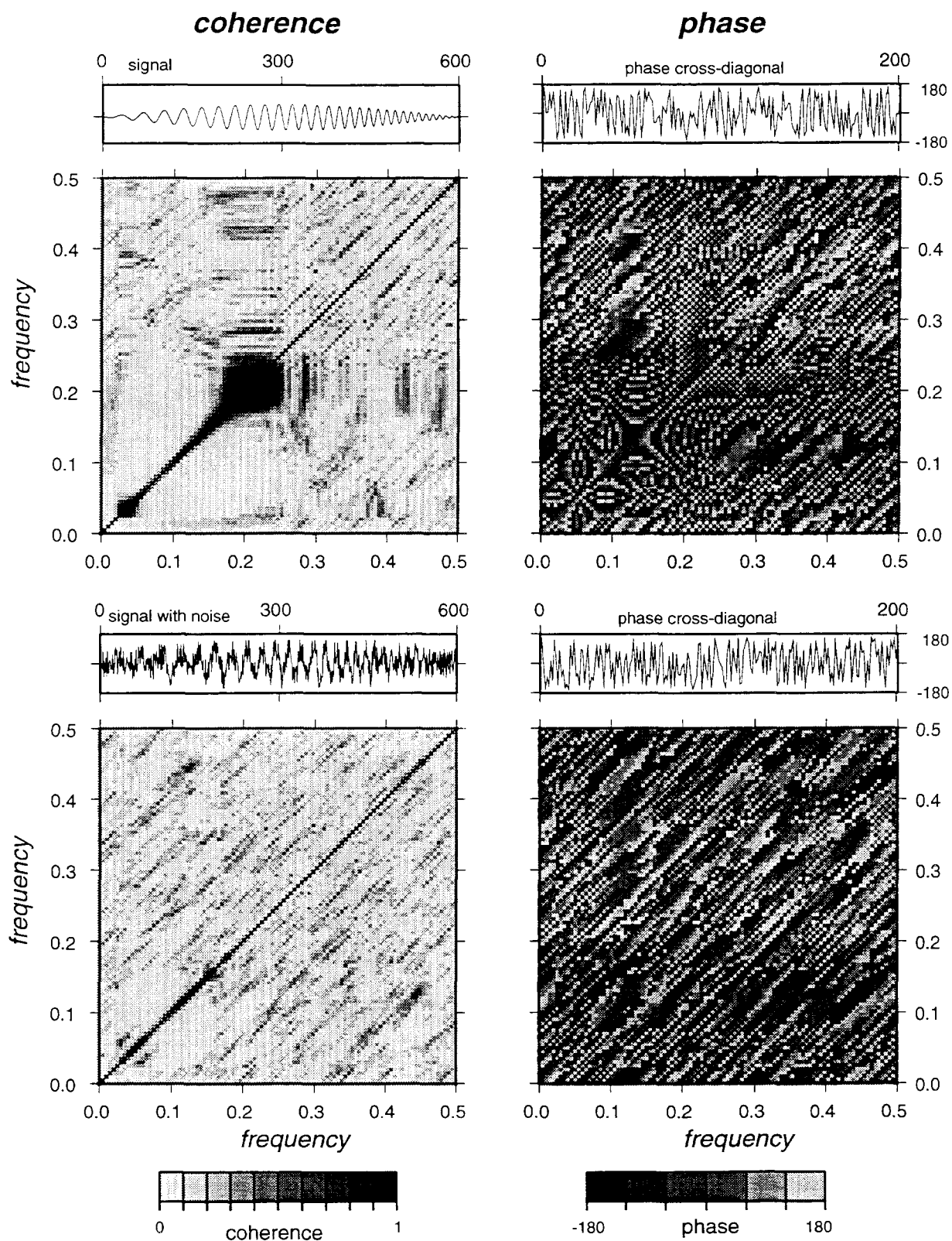
coherence

phase

Figure 1. Double coherence of a sweep signal ranging in frequency from 0.03 Nyquist to 0.2 Nyquist. Pure signal (top) and signal with noise (bottom) are shown. Signals are shown on left above corresponding matrix plots. Phase cross-diagonals are shown on left above matrix plots.
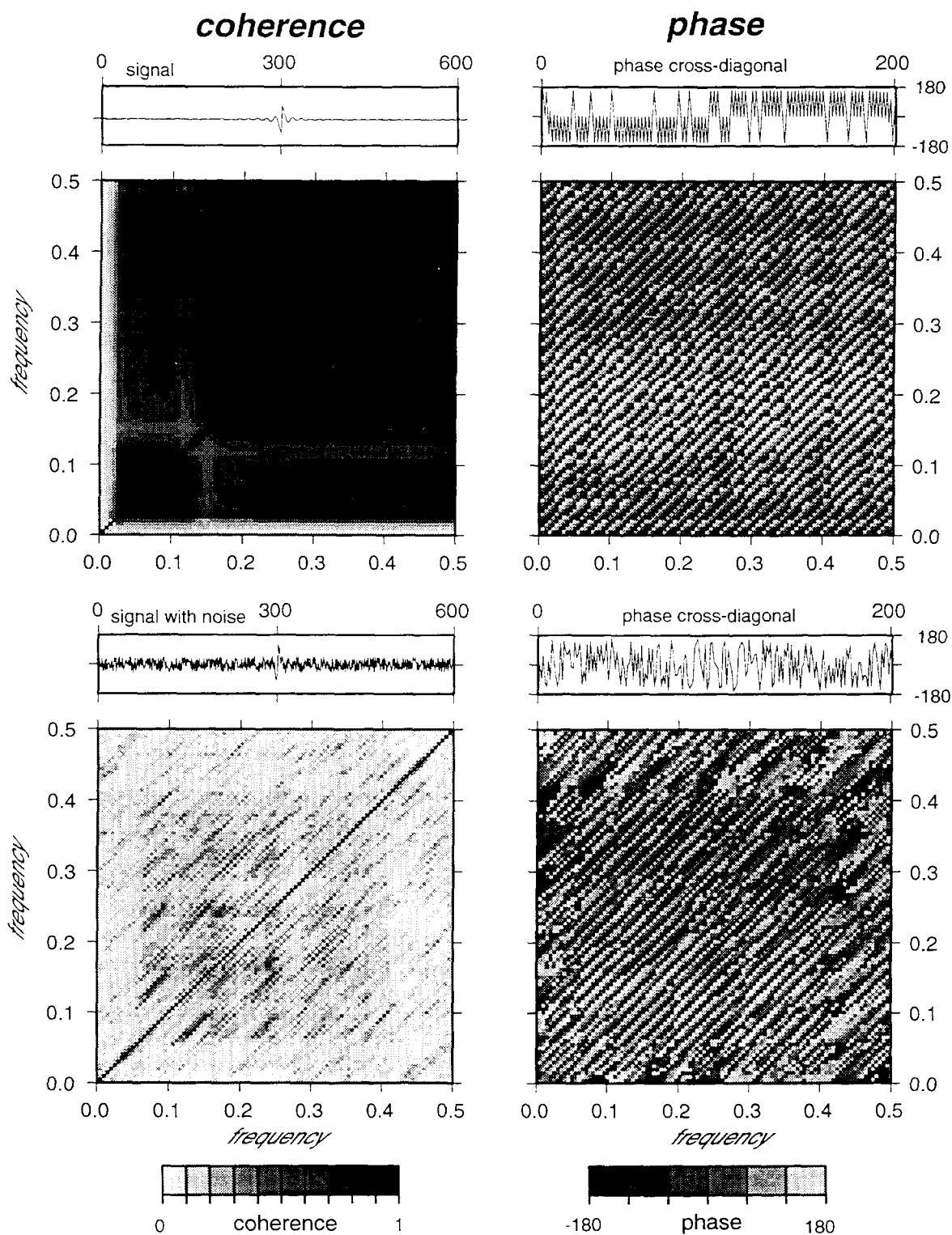
Figure 2. Double coherence and phase of a 0.15 Nyquist sinc-like function with and without noise. High coherence of the pure signal is greatly reduced by noise except at near the primary frequency. Note the clear phase relationship shown by the phase plots.

coherence

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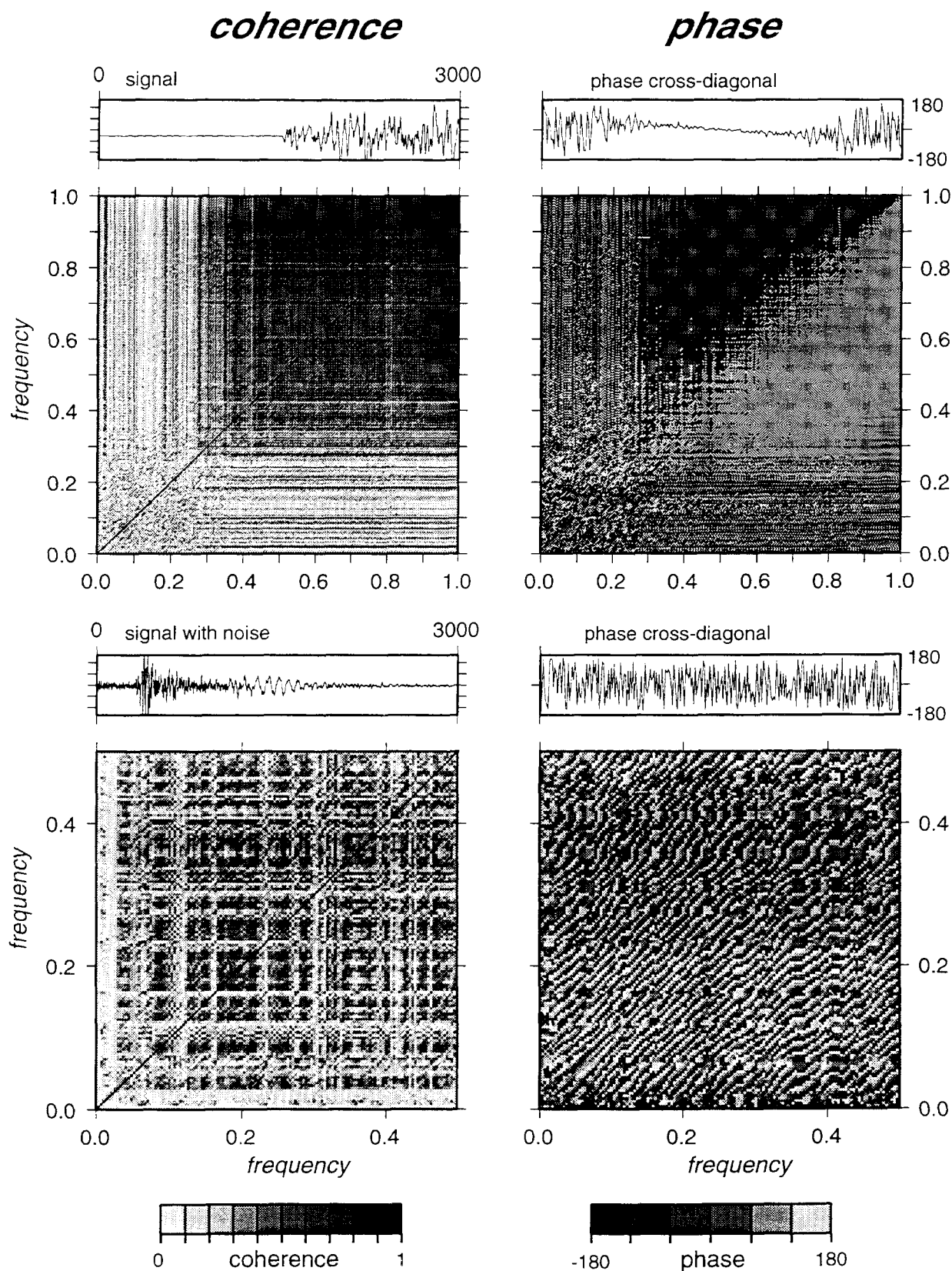
Figure 3. Double coherence for broadband seismic data recorded in Central Asia by the Kyrgyzstan broadband network. Event at top is teleseismic P wave and lower event is surface waves from a regional (300 km) earthquake. Note that the upper plots show full Nyquist frequency.